Regularized Learning in Harmonic Games

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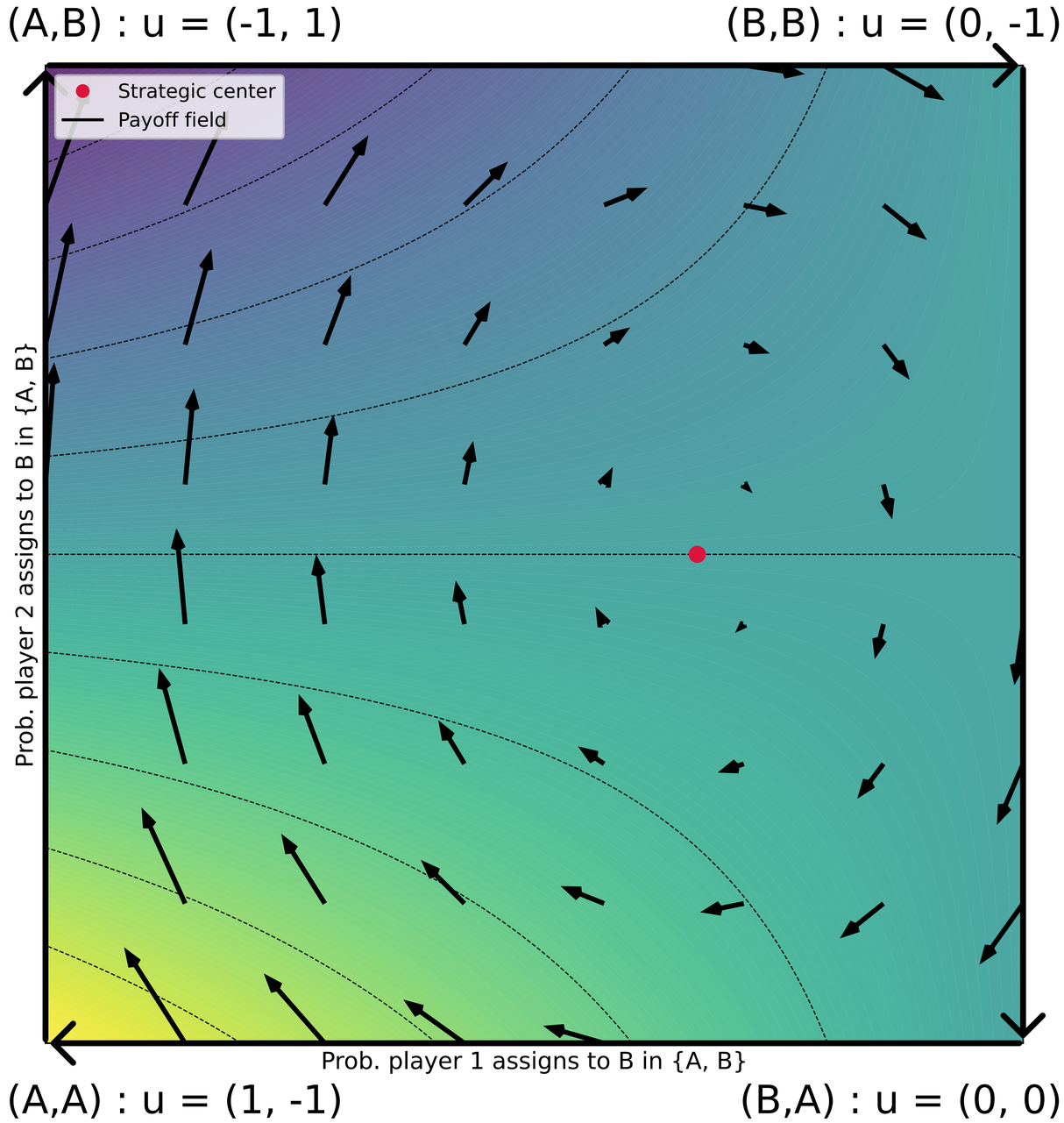
Harmonic games and where to find them

Harmonic games

→ Conflicting interests

→ Strategic center



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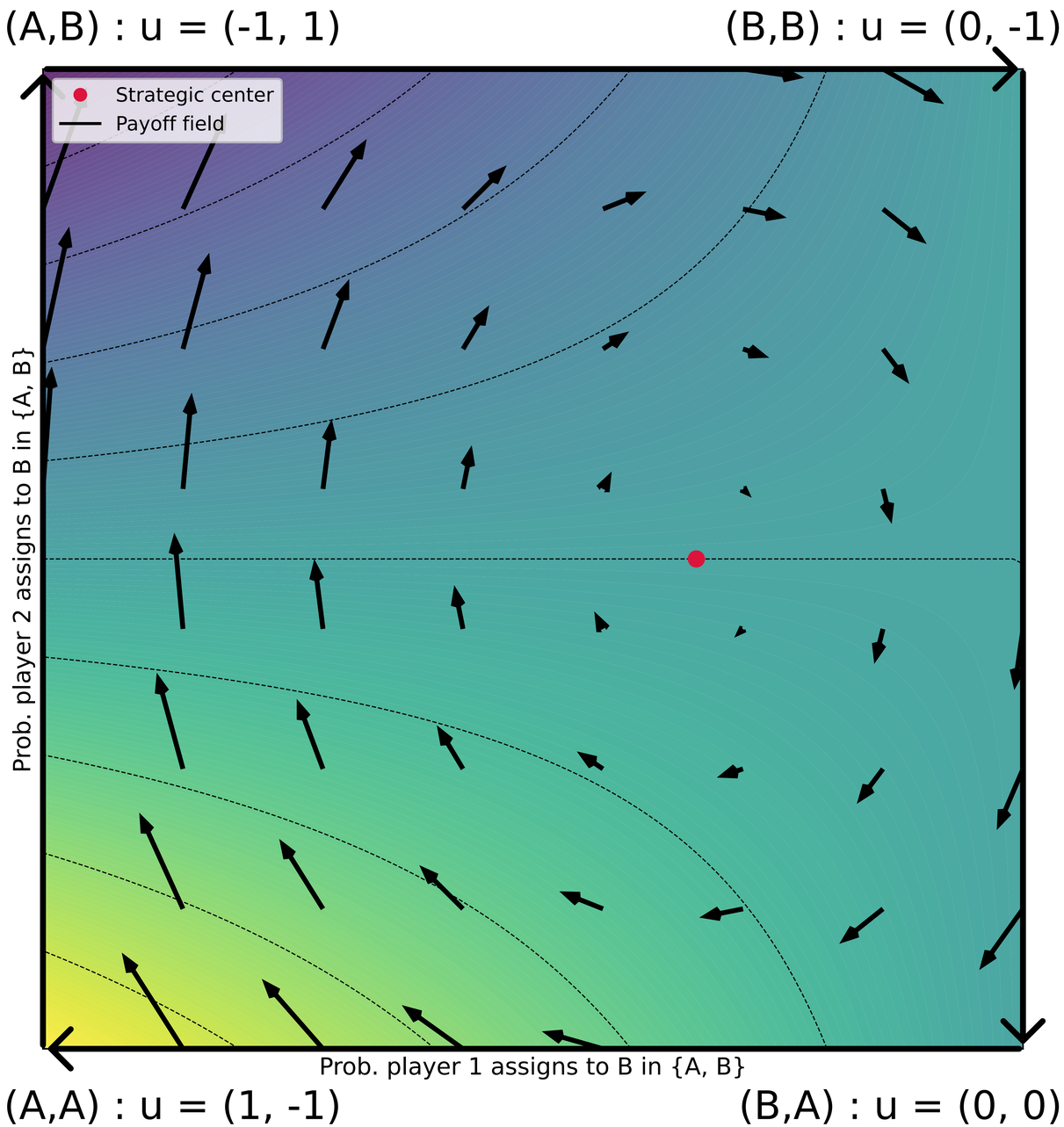
→ Strategic complement of potential games

$$\Gamma = \Gamma_{pot} + \Gamma_{harm}$$



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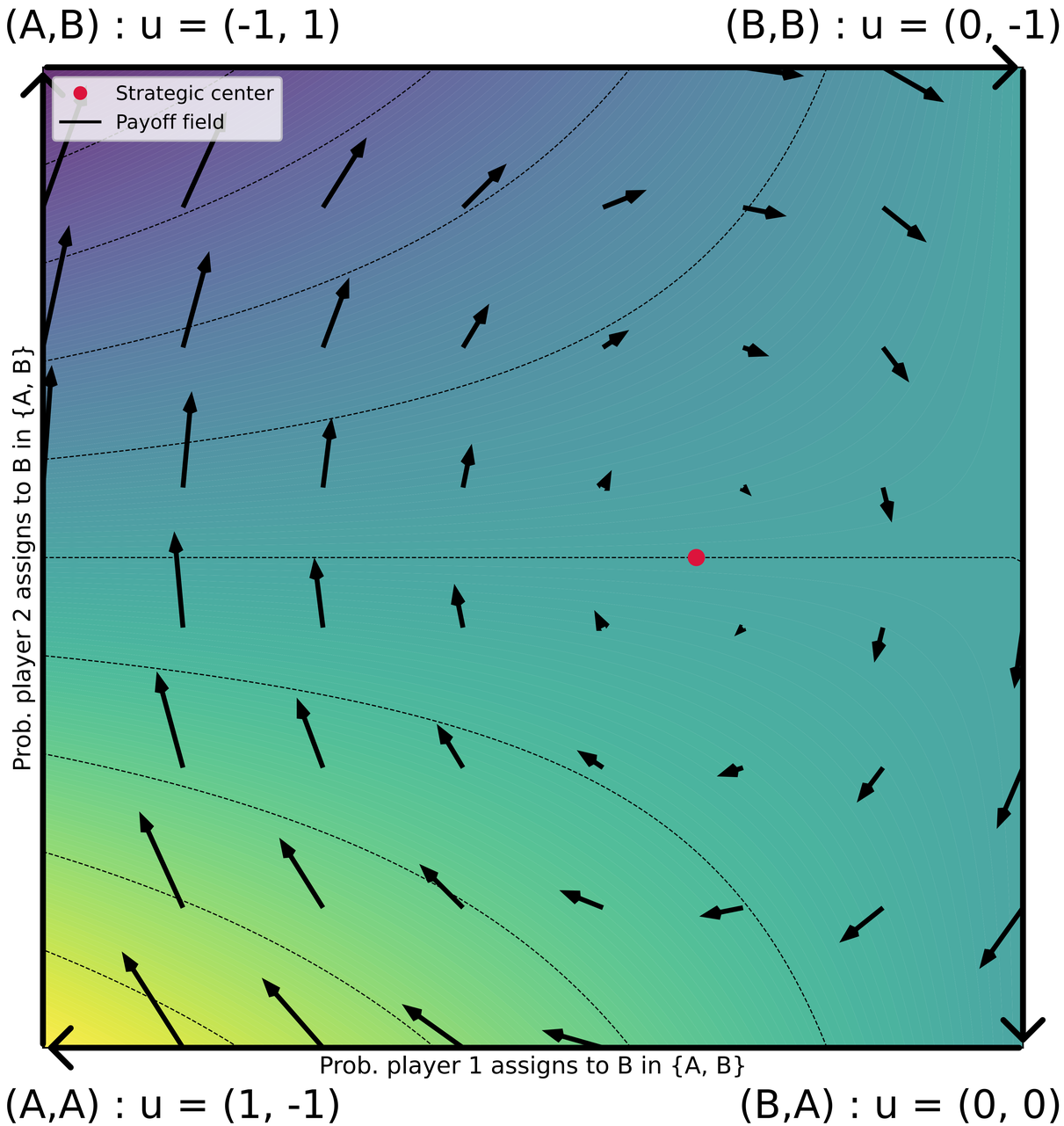
Dynamics?



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Follow The Regularized Leader

$$\dot{y} = V(x)$$
$$x = Q(y)$$
$$Q(y) = \text{regularized argmax}$$

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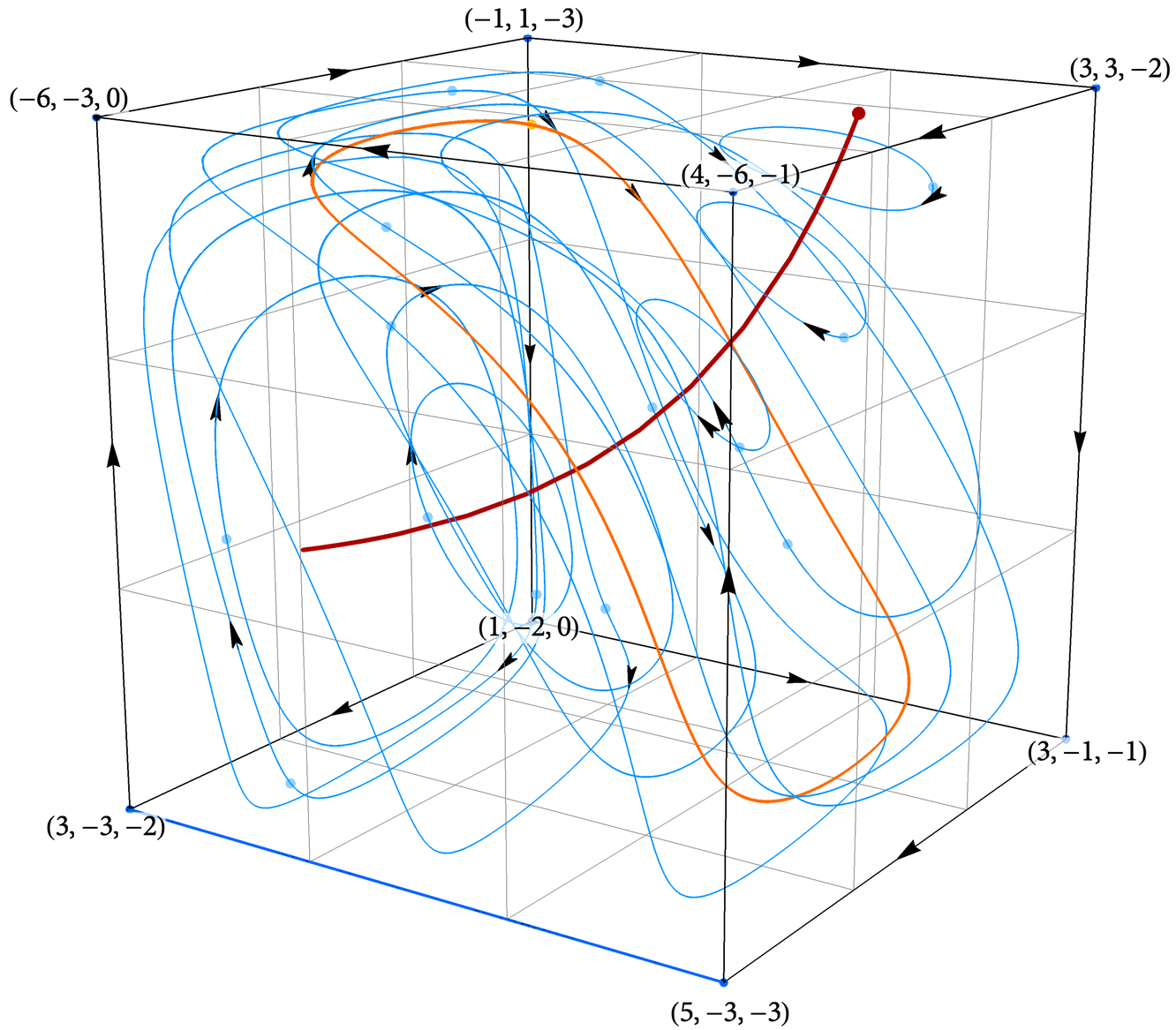
Theorem

*In continuous time, FTRL in harmonic games is recurrent**

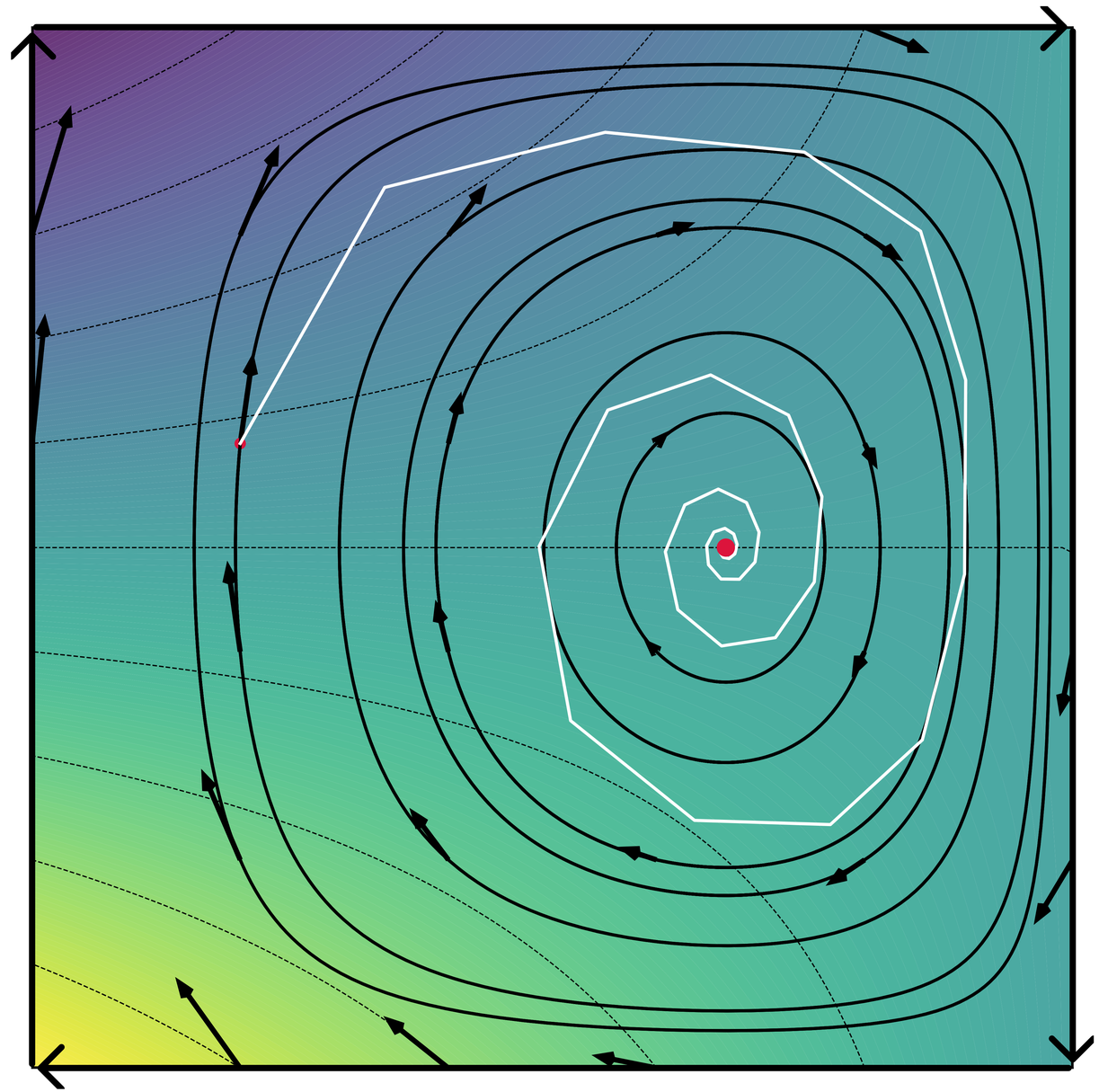
** Read: quasi-periodic*



Discrete time: optimism in the face of recurrence

Follow The Regularized Leader +

$$y_* = y + \eta\, \hat{V}$$
$$y_+ = y + \eta\, V(x_*)$$
$$x = Q(y)$$



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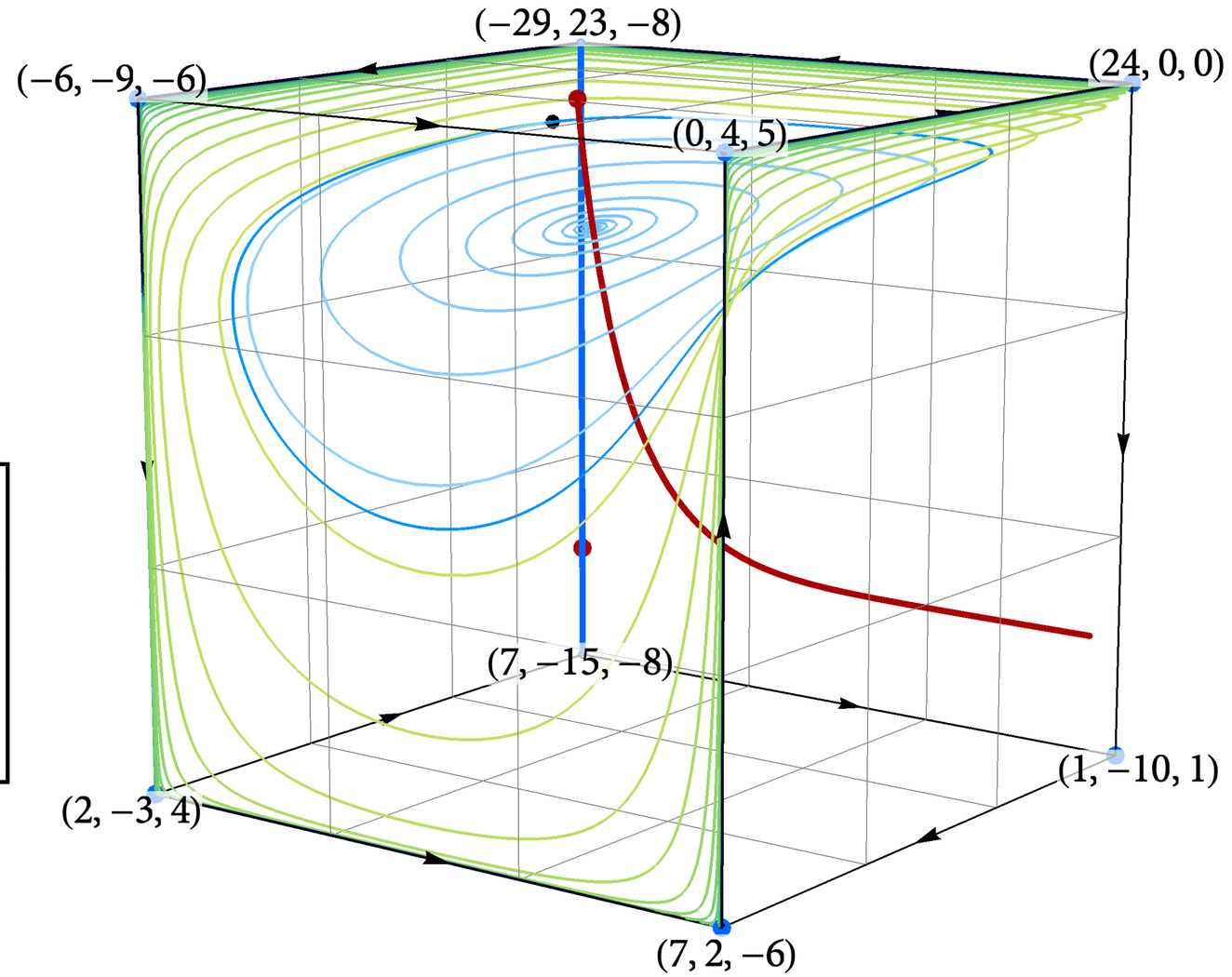
In discrete time, FTRL+ in harmonic games converges to a Nash equilibrium, and each player is guaranteed constant regret



Positive convergence result in new class of games. So what?

→ *Harmonic games* ***complement potential games*** *strategically* ***and dynamically***

→ *Extend existing results for two-player zero-sum games to* ***non-convex setting***

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Thanks!