A Geometric Approach to Game Dynamics

What is the Geometric Obstacle to Convergence?

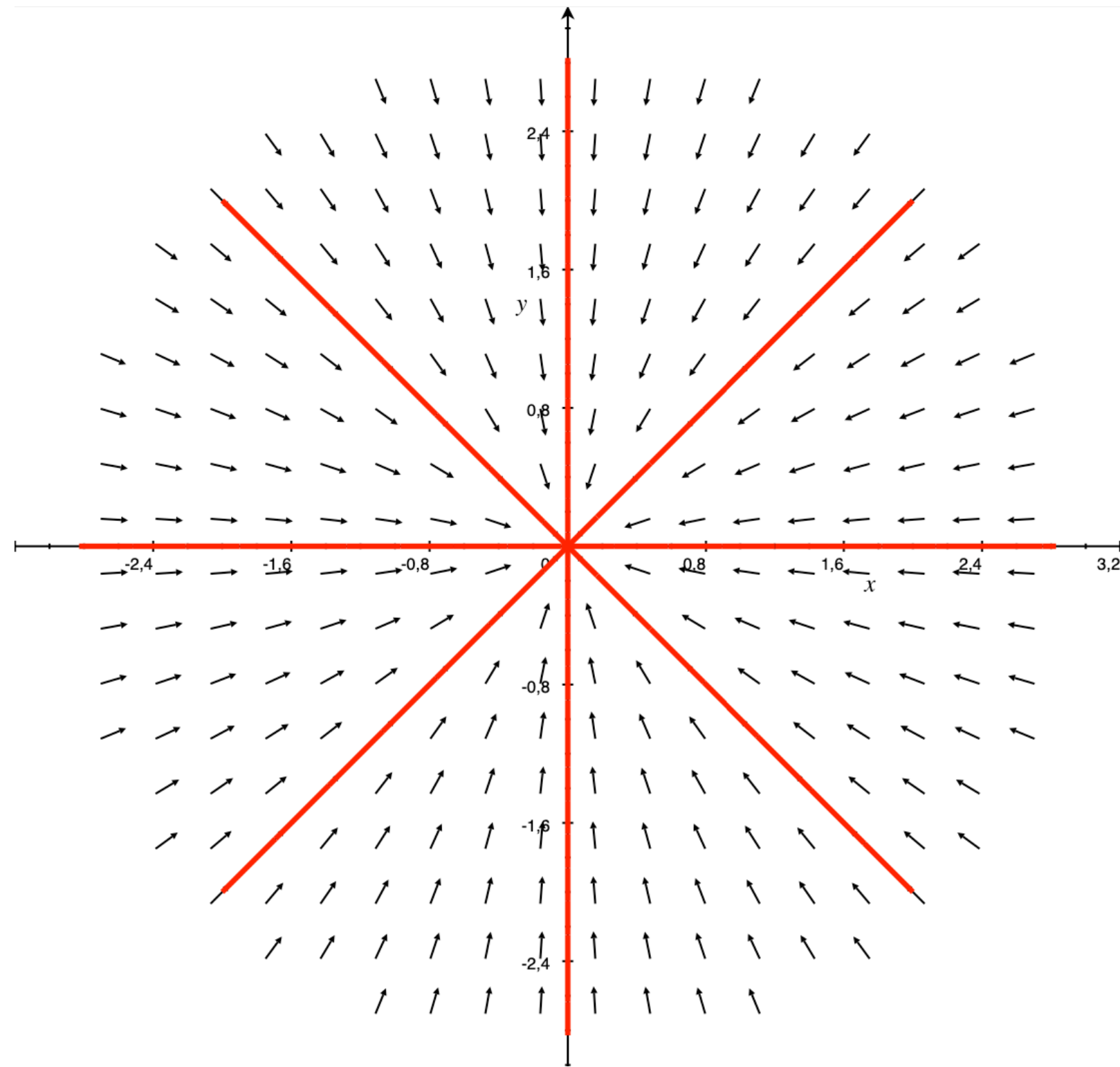**Davide Legacci**, Panayotis Mertikopoulos, Bary Pradelski

Univ. Grenoble Alpes, CNRS, Inria, Grenoble INP, LIG, 38000 Grenoble, France
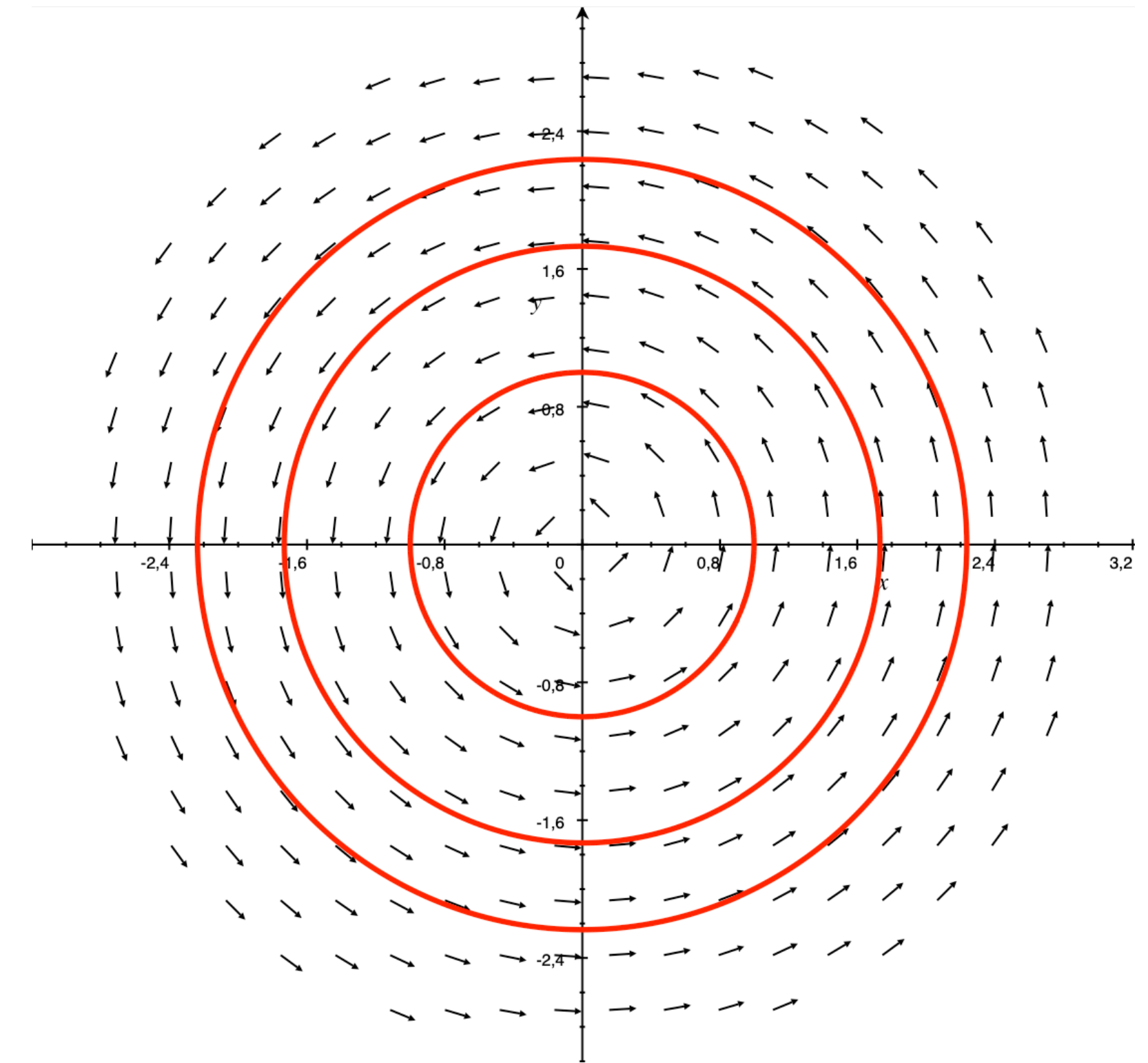
June 2023

An image from physics

The flows of...

An <u>electric</u> field

- Converging
- Rotation-free
- **Scalar** potential $E = \nabla f$

A <u>magnetic</u> field
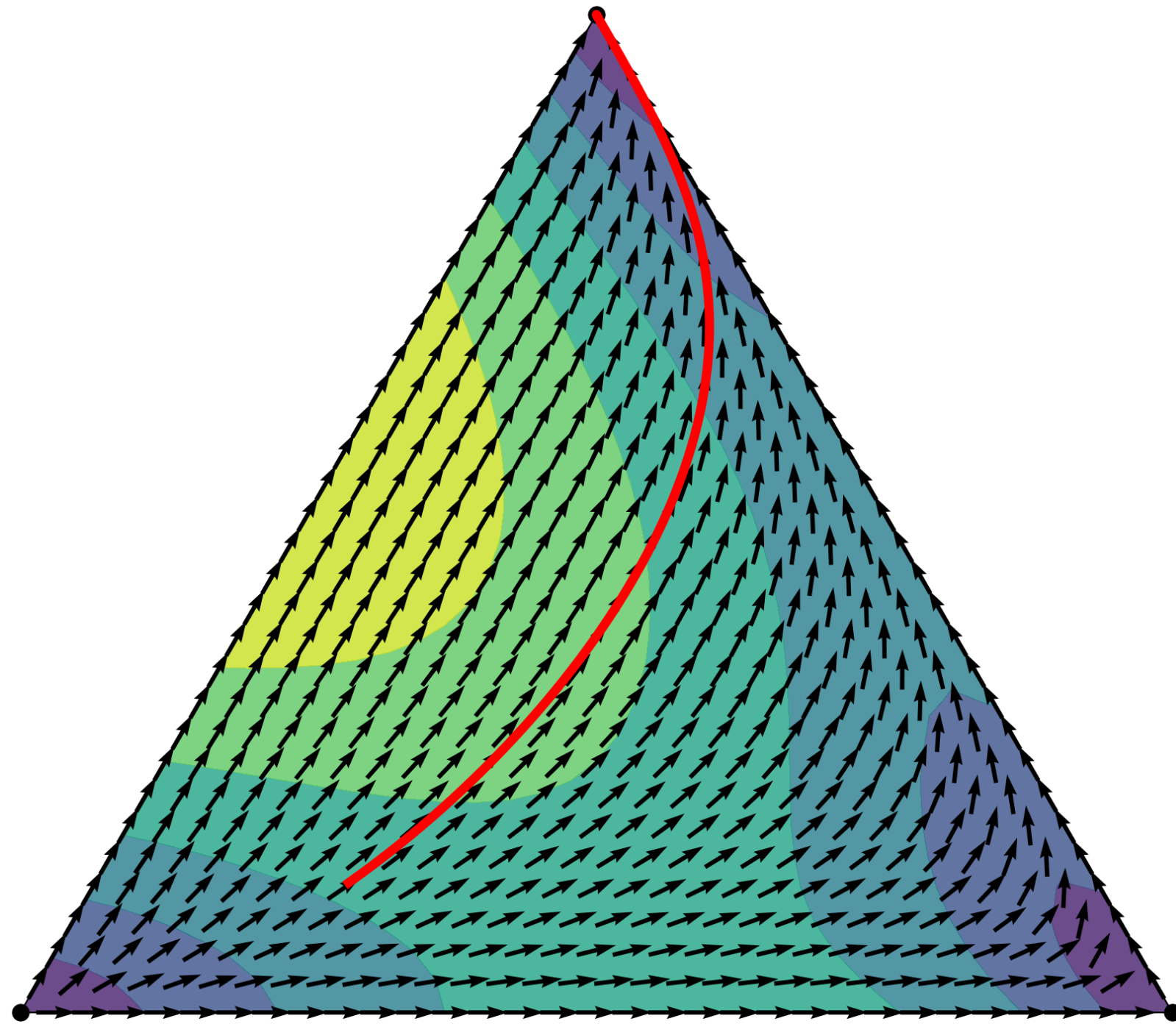

- Conservative
- Divergence-free
- **Vector** potential $B = \nabla \times A$
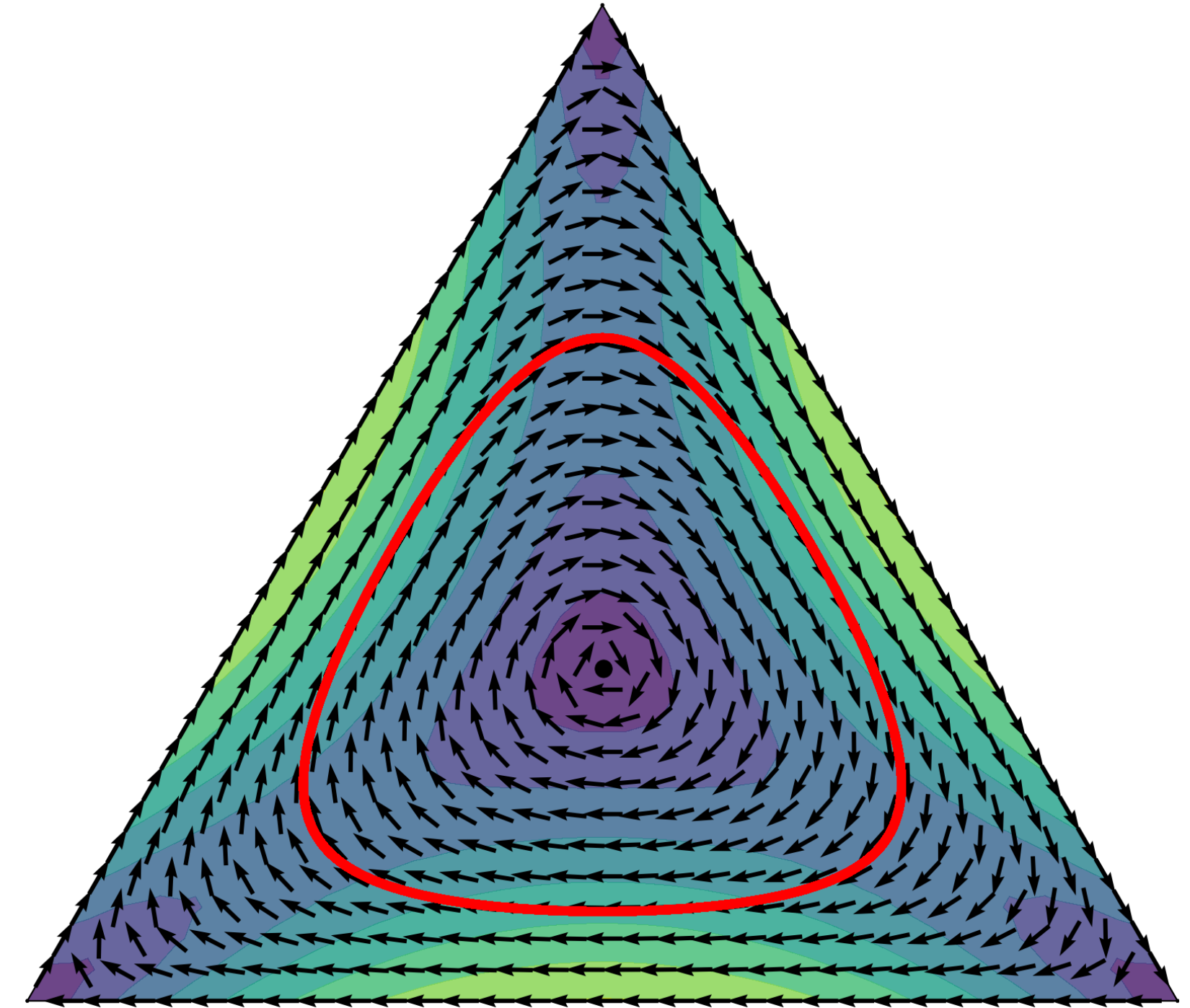
An image from game theory

Replicator dynamics in…

A <u>potential</u> game

A **co-exact** game*

- Converging
- Rotation-free $dv = 0$
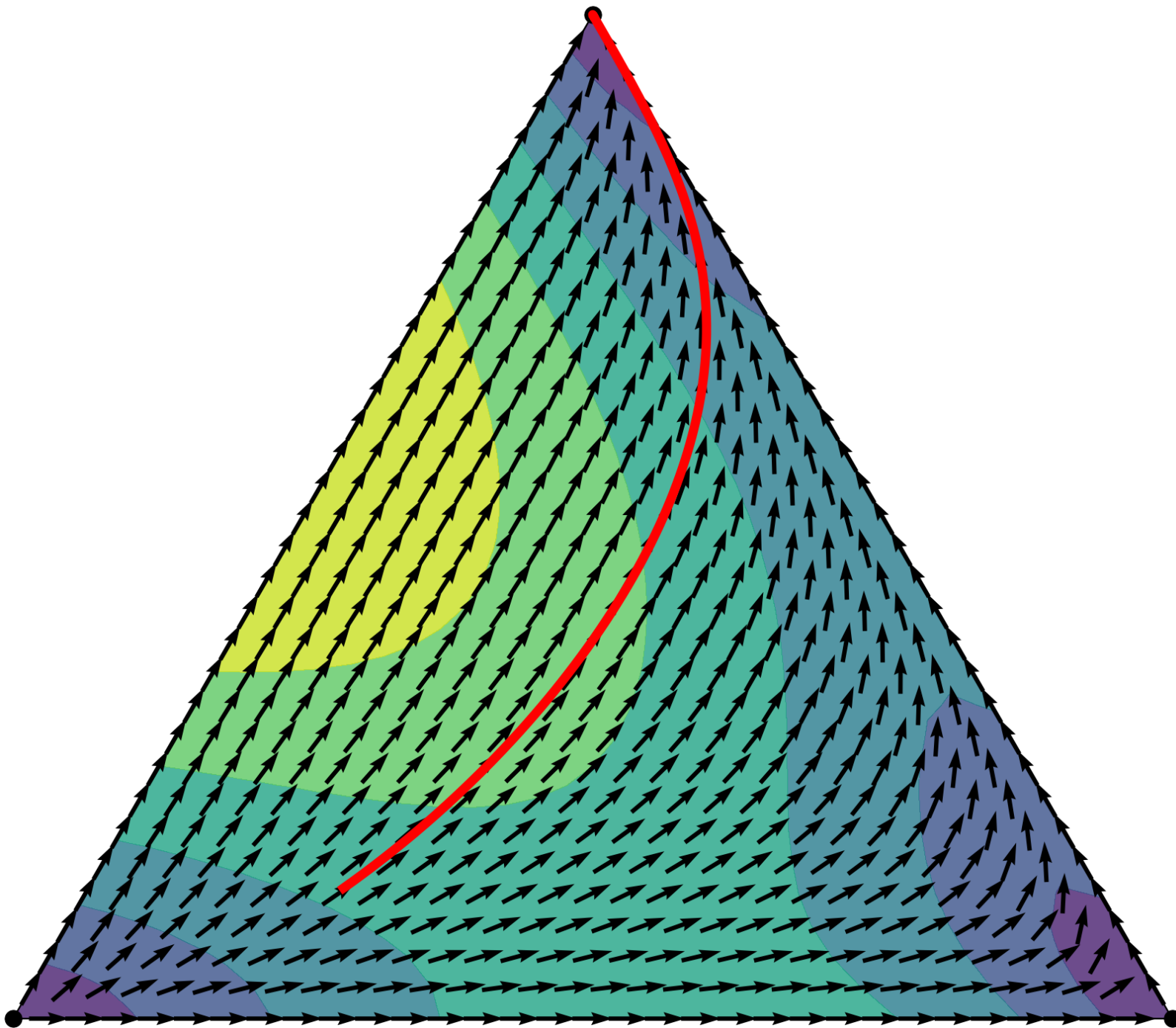- **Scalar** potential $v = df$

- Conservative
- Divergence-free $\delta v = 0$
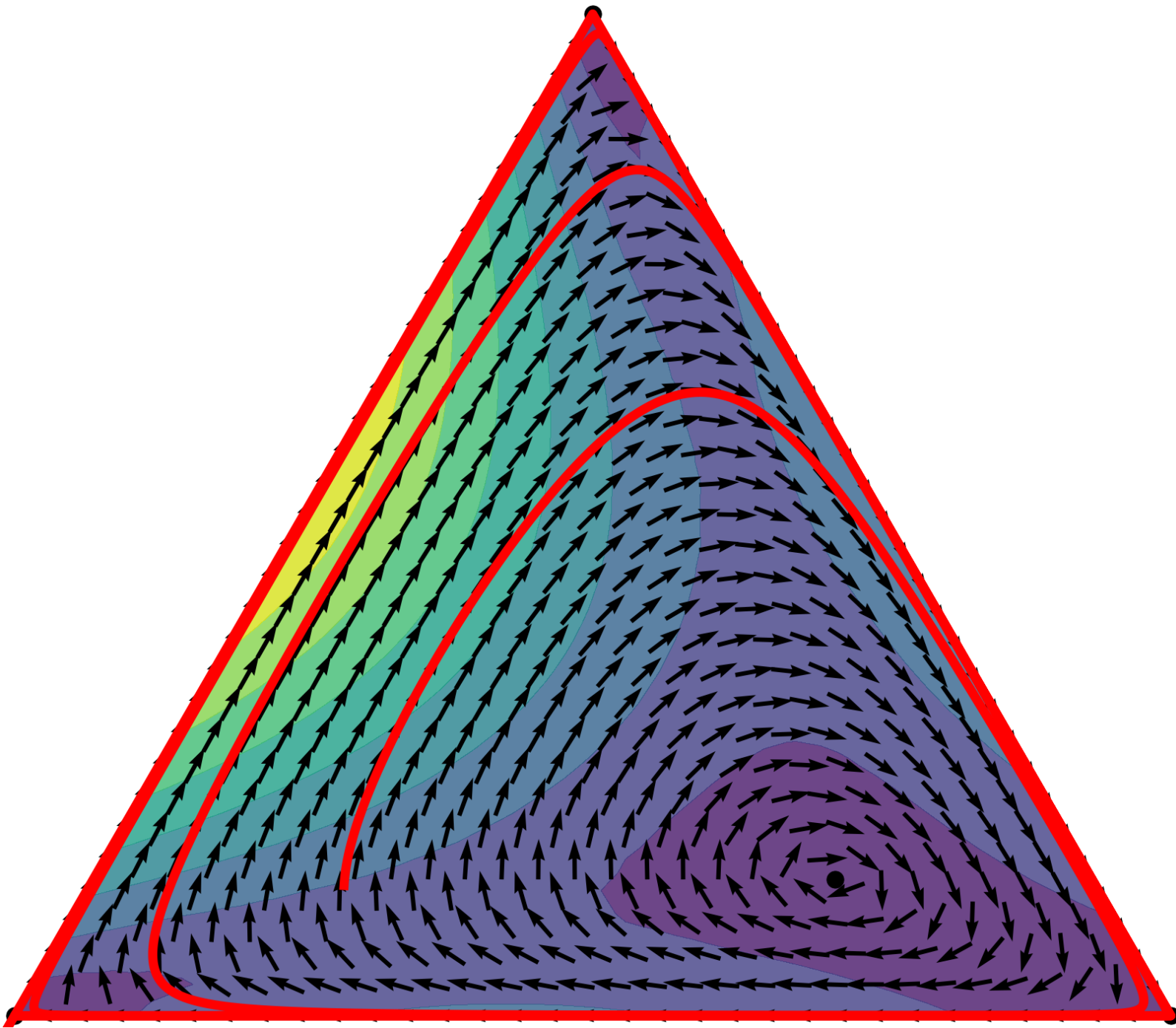- **Tensor** potential $v = \delta A$

An image from game theory

Replicator dynamics in...

A <u>potential</u> game

A generic game

A <u>**co-exact**</u> game*

- Converging
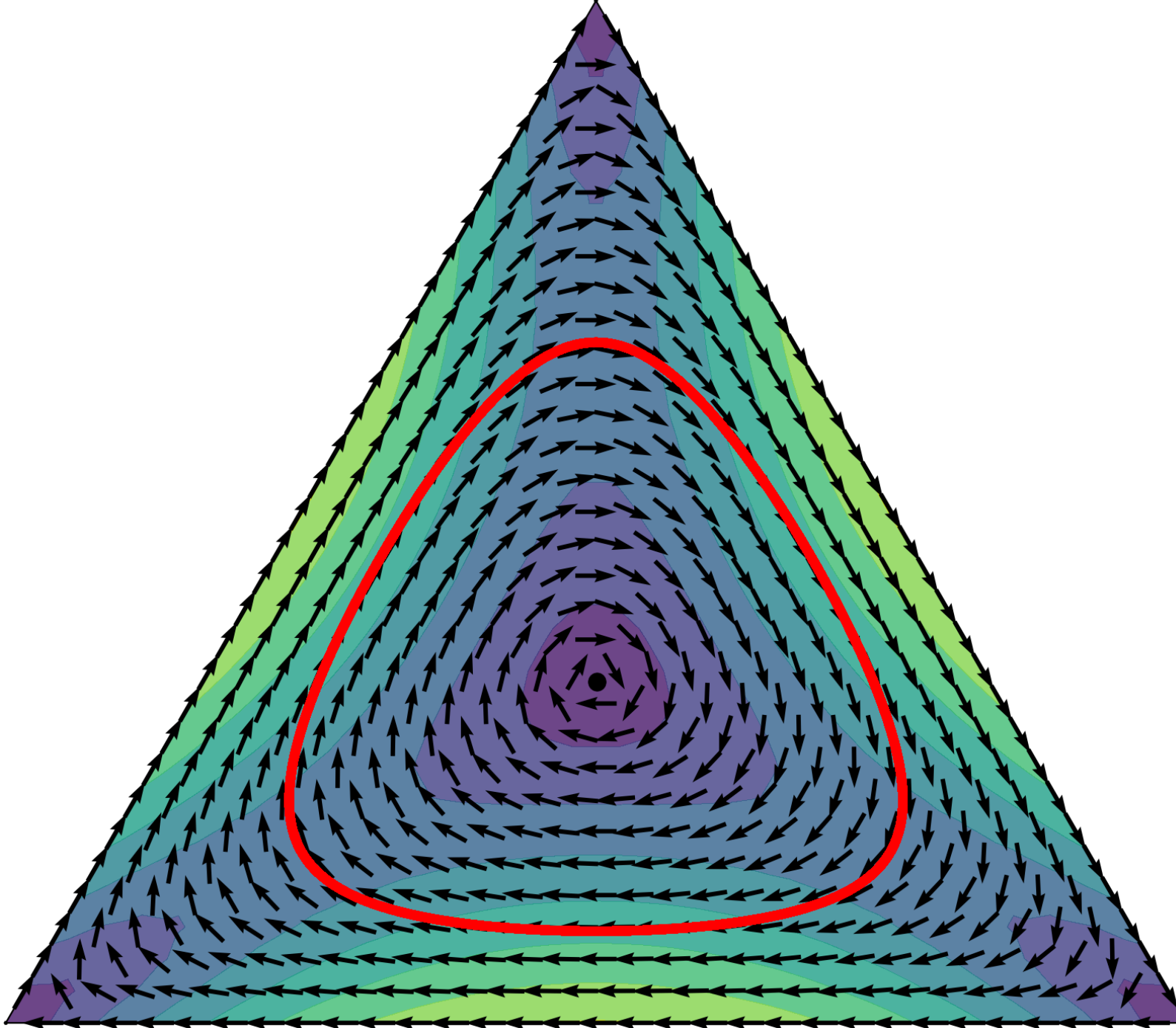- Rotation-free $dv = 0$
- **Scalar** potential $v = df$

Payoff Hodge decomposition
$$v = df + \delta A$$

Co-exactness*
is the obstacle to convergence

- Conservative
- Divergence-free $\delta v = 0$
- **Tensor** potential $v = \delta A$

*δ = Shahshahani co-differential
#AMA in the poster session